Polarimetric neutron scattering

F. Tasset¹

¹ Institut Laue-Langevin, BP. 156, 38042 Grenoble Cedex 9, France

Abstract. Beams of spin polarized itinerant neutrons have long been used to map magnetic electrons in ferromagnetic and paramagnetic materials under an applied field. End of 1960's, Moon, Riste and Koehler introduce spin resolved neutron scattering at HFIR in Oak-Ridge, enlarging considerably the field of polarized neutrons. But it is only during the last decade, with the introduction of the zero-field polarimeter Cryopad at ILL, that the 3 components of the neutron spin polarization vector are measured independently before and after scattering, giving the neutron all its strength to uncover complicated antiferromagnetic arrangements. Our slides ought to introduce the spin polarization vector, the two main neutron scattering amplitudes, magnetic and nuclear, the high sensitivity which results from their interference, and the experimental techniques available in Grenoble for such measurements.

1 THEORY

The Angular momentum operator $\vec{J}$

- Quantum definition of angular momentum operator

- In quantum mechanics the concept of angular momentum evolves from the rotation operators (matrices which transform the amplitudes in between each other)

$$|a\rangle \xrightarrow{\text{unitary, linear}} |a'\rangle$$

$$|a'\rangle = \mathbf{R}|a\rangle$$

$$\mathbf{R}(\vec{u}, \delta\omega) = 1 - i \delta\omega \vec{u} \cdot \vec{J}$$

- On a particle without spin

$$\psi'(\vec{x}) = \psi(\mathbf{R}^{-1}\vec{x}) = \mathbf{R} \psi(\vec{x})$$

- It can be shown that the z component

$$J_z = \frac{1}{i} \frac{\partial}{\partial \varphi}$$

- And more generally that the $\vec{J}$ vector operator is looking like the classical definition of the angular momentum;

$$\hbar \vec{J} = \vec{x} \wedge \vec{p}$$

Conservation of $\vec{J}$

1.1 Conservation of angular momentum

- For a rotation invariant system the energy must not depend on rotation
- Therefore, due to preceding definition of $\vec{J}$ we must have

$$[H, R] = 0$$

$$R(\vec{u}, \delta\omega) = 1 - i \delta\omega \vec{u} \cdot \vec{J}$$

$$[H, \vec{J}] = 0$$

- And a Common eigen basis

$$H |\alpha, j, m\rangle = E_{\alpha, j} |\alpha, j, m\rangle$$

$$J_z |\alpha, j, m\rangle = m |\alpha, j, m\rangle$$

$$\vec{J}^2 |\alpha, j, m\rangle = j(j+1) |\alpha, j, m\rangle$$

2π Periodicity & Quantification of $\vec{J}$

Possible values of angular momentum

- The groups of centred rotations in laboratory space R and the group of quantum mechanical rotations operators R (matrices) must be isomorphic
- Therefore the irreducible representations of R_3 and the corresponding base vectors constitute a complete set which is appropriate to express any spin quantum state vector and to express how it transforms under space rotations
- The set of basis vectors is not continuous. It is discrete due to 2π periodicity in R .
- For a conservative system. One given j set is globally invariant (irreducible)

$$R_1 \bullet R_2 = R_3$$

$$R_1 \bullet R_2 = R_3$$

$$R |\alpha, j, m\rangle = \sum_{m'} |\alpha, j, m'\rangle \langle m' | D_j(R) | m \rangle$$

$$j = \text{integer or } \frac{1}{2} \text{ integer}$$

$$m = j, j-1, \dots, -j$$

For The Neutron Spin S, J=1/2

- Spin 1/2 : 2D Hilbert Space**

The Hilbert space for spin 1/2 is 2 dimensional

$$j = \frac{1}{2} \quad 2j + 1 = 2$$

$$|\chi\rangle = \alpha \left| \frac{1}{2} \right\rangle + \beta \left| -\frac{1}{2} \right\rangle$$

- 3D-angular momentum vector operator**

- The expectation value of the 3 components of angular momentum

$$s_x = \frac{1}{2} \hbar \langle \hat{\sigma}_x \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_x | \chi \rangle$$

$$s_y = \frac{1}{2} \hbar \langle \hat{\sigma}_y \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_y | \chi \rangle$$

$$s_z = \frac{1}{2} \hbar \langle \hat{\sigma}_z \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_z | \chi \rangle$$

- The 3 Pauli Matrices

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \hat{\sigma}_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

"Horizontal" Neutron Spin State

- Non trivial example of 3D-angular momentum components

$$|\chi\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \right\rangle + \frac{1}{\sqrt{2}} \left| -\frac{1}{2} \right\rangle$$

- It is enough to have equal moduli for quantum amplitudes to be in "up" and "down" to get the momentum in the "horizontal" plane

- Zero angular momentum in z direction

$$s_z = \frac{1}{2} \hbar \langle \hat{\sigma}_z \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_z | \chi \rangle = \frac{1}{2} \hbar \left| \frac{1}{\sqrt{2}} \right| \cdot \left| \frac{1}{\sqrt{2}} \right| \cdot \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 0$$

- All along x direction

$$s_x = \frac{1}{2} \hbar \langle \hat{\sigma}_x \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_x | \chi \rangle = \frac{1}{2} \hbar \left| \frac{1}{\sqrt{2}} \right| \cdot \left| \frac{1}{\sqrt{2}} \right| \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{2} \hbar$$

$$s_y = \frac{1}{2} \hbar \langle \hat{\sigma}_y \rangle = \frac{1}{2} \hbar \langle \chi | \hat{\sigma}_y | \chi \rangle = \frac{1}{2} \hbar \left| \frac{1}{\sqrt{2}} \right| \cdot \left| \frac{1}{\sqrt{2}} \right| \cdot \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = 0$$

Beam statistics: Density matrix

- *Beam mean value : the statistical average on an ensemble of neutrons*

$$\langle \hat{A} \rangle = N^{-1} \sum_{k=1}^N \int \psi^{k*} \hat{A} \psi^k d\tau$$

$$\psi^k = \sum_n c_n^k \varphi_n, (k = 1, \dots, N)$$

- *Density matrix: a matrix which describes the mean value of "quantum amplitudes products"*
 - *Diagonal elements are just the probability to be in a given basis state, it is normalised and hermitian.*
- *The Beam mean value is easy to express with such matrices*

$$A_{mn} = \int \varphi_m^* \hat{A} \varphi_n d\tau$$

$$\rho_{mn} = N^{-1} \sum_k c_n^{k*} c_m^k$$

$$\langle \hat{A} \rangle = \sum_{m,n} A_{mn} \rho_{nm} = \text{Tr}(\hat{\rho} \hat{A})$$

Defining the spin polarisation vector

- *The density matrix describes the statistical quantum situation fully.*
- *For spin 1/2, it is a 2*2 hermitian complex normalised matrix: 3 real numbers define it completely.*
- *3 independent $\langle A \rangle$ measurements are needed.*
- *Using polarimetric tools we can measure the normalised angular momentum $P = (n^+ - n^-)/(n^+ + n^-)$ in 3 orthogonal directions x, y, z, in turn. We call them P_y, P_x, P_z . That is enough to describe the situation fully.*

$$\langle A \rangle = N^{-1} \sum_{k=1}^N \int \psi^{k*} \hat{A} \psi^k d\tau$$

$$\langle A \rangle = \sum_{m,n} A_{mn} \rho_{nm} = \text{Tr}(\hat{\rho} \hat{A})$$

$$\vec{P} = \langle \vec{\hat{\sigma}} \rangle$$

$$\hat{\rho} = a \cdot \hat{1} + (\vec{a} \cdot \vec{\hat{\sigma}})$$

$$\vec{P} = \langle \vec{\hat{\sigma}} \rangle = \text{Tr} \hat{\rho} \vec{\hat{\sigma}} = \text{Tr} [a \vec{\hat{\sigma}} + \vec{\hat{\sigma}} (\vec{a} \cdot \vec{\hat{\sigma}})] = 2\vec{a}$$

$$\hat{\rho} = \frac{1}{2} [\hat{1} + (\vec{P} \cdot \vec{\hat{\sigma}})] = \frac{1}{2} \begin{bmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{bmatrix}$$

Introducing the magnetic moment

- *Neutron magnetic moment*
 - The dipole moment is a vector
 - Rotation group theory tells us that it has to be proportional to angular momentum vector

$$\mu_{neut} = g_N \mu_N = -1.9130427(5) \frac{m_e}{m_p} \mu_B$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2741 \cdot 10^{-24} \text{ JT}^{-1} (\text{SI})$$

$$\mu_{neut} = 9.6622 \cdot 10^{-27} \text{ JT}^{-1} (\text{SI})$$

- *Zeeman energy term*
 - Classical expression
 - Quantum expression

$$H_s = -\vec{\mu} \cdot \vec{B}$$

$$H_s = -\mu_{neut} \vec{\sigma} \cdot \vec{B}$$

Eq. of movement for Polarisation

- *Quantum equation of evolution for a mean value:*

$$\frac{\partial \vec{P}}{\partial t} = \frac{\partial \langle \vec{\sigma} \rangle}{\partial t} = \frac{1}{\hbar} \langle [\vec{\sigma}, H_s] \rangle$$

- *& the neutron magnetic moment:*

$$H_s = -\gamma \frac{\hbar}{2} (\vec{\sigma} \cdot \vec{B})$$

=

$$\frac{\partial \vec{P}}{\partial t} = -\gamma \langle (\vec{B} \wedge \langle \vec{\sigma} \rangle) \rangle$$

A handle on its spin polarisation

$$\frac{\partial \vec{P}}{\partial t} = -\gamma \langle (\vec{B} \wedge \vec{P}) \rangle$$

$$\gamma = -183.2461 \cdot 10^6 \text{ s}^{-1} \text{ T}^{-1}$$

Solution in a constant mag-field

- Larmor Precession of the polarisation vector

$$\frac{\Delta\varphi}{\Delta x} [\text{deg/cm}] = 2.65 \lambda [\text{\AA}] H [\text{Gauss}]$$

- A neutron with a 2.4 \AA wavelength precesses by 60° in a 10 Gauss.cm field integral.

Solution in a turning mag-field

- The polarisation vector in a reorienting guide field

- Depends on adiabaticity parameter

$$E = \frac{\omega_L}{\omega_H}$$

Components of P as a function of the adiabaticity parameter

- For large E, the "longitudinal" component rotates adiabatically with field. Zeeman energy is conserved.
- For small E, the field is abruptly flipped, the polarisation direction stays as it was. Small change in Zeeman energy (60.31 neV/Tesla)

Handling Tools

- The neutron Cryoflipper

Handling Tools

- The Nutator: adiabatic rotation of a guide field

Handling Tools

- Larmor precession in Cryopad:

Handling Tools

- + Adiabatic nutators
- + Larmor precession coils
- + Field free chamber

Cryopad

Neutron Scattering

- **Scattering amplitude : Fermi golden rule,**
 - Time dependent Schrödinger equation
 - In a perturbative approach the proba to go from state $|a\rangle$ to state $|b\rangle$ in a given time t
 - In neutron scattering we have a degenerate continuum of final states accessible $D(E_f)$ with the same energy as the initial. The Fermi golden rule gives such a probability to go from state $|a\rangle$ to state $|b\rangle$ per unit of time.
 - It results in a elastic scattering cross section

$$\hbar i \frac{d}{dt} |u(t)\rangle = H |u(t)\rangle$$

$$w_{b \leftarrow a}(t) = \frac{1}{\hbar} \int_0^t dt' e^{i\omega_{ba}t'} \left| \langle b | H^{(1)}(t') | a \rangle \right|^2$$

$$W_{ji} = \frac{2\pi}{\hbar^2} \left| \langle j | H^{(1)} | i \rangle \right|^2 \delta(\omega_{ji})$$

$$\frac{dW}{d\Omega} = \frac{V}{(2\pi)^2} \frac{m}{\hbar^3} k_0 \left| \langle \vec{k}' | V | \vec{k}_0 \rangle \right|^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2} \frac{m^2}{\hbar^4} \left| \int d^3\vec{x} e^{i(\vec{k}_0 - \vec{k}) \cdot \vec{x}} V(\vec{x}) \right|^2$$

Scattering potentials

- **Scattering cross sections:**

$$\frac{d^2\sigma}{d\Omega dE} = \left(\frac{m}{2\pi\hbar^2} \right)^2 \frac{k'}{k} \sum_{\lambda, \lambda'} p_{\lambda} \text{Tr} \hat{\rho} \left(\lambda' \left| \hat{V}(\kappa) \right| \lambda \right) \left(\lambda \left| \hat{V}(\kappa) \right| \lambda' \right) \delta(\hbar\omega + E_{\lambda} - E_{\lambda'})$$

- **Example of scalar interaction operators**

$$\hat{V}_N(\kappa) = \frac{2\pi\hbar^2}{m} \sum_{l,d} \exp(i\kappa \cdot \mathbf{R}_{ld}) \hat{b}_{ld}$$

- **Example of vector interaction operators**

$$\hat{V}_M(\kappa) = \left(\frac{2\pi\hbar^2}{m} \right) \left(\frac{\gamma e^2}{m_e c^2} \right) \hat{\sigma} \cdot \sum_i \exp(i\kappa \cdot \mathbf{r}_i) \left[\vec{\kappa} \times (\hat{\mathbf{s}}_i \times \vec{\kappa}) - \frac{i}{\hbar|\kappa|} \vec{\kappa} \times \hat{\mathbf{p}}_i \right]$$

- **They are all of the following kind**

$$\hat{V} = \hat{\beta} + \vec{\hat{\alpha}} \cdot \vec{\hat{\sigma}}$$

Scattered Spin Polarisation Vector

- Maleyev, Blume, Lowesey

$$\mathbf{P}' = \text{Tr} \hat{\rho} \hat{v}^* \hat{\sigma} \hat{v} / \text{Tr} \hat{\rho} \hat{v}^* \hat{v}$$

$$\text{Tr} \hat{\rho} \hat{v}^* \hat{v} = \hat{\alpha}^* \hat{\alpha} + \hat{\beta}^* \hat{\beta} + \hat{\beta}^* (\hat{\alpha} \cdot \mathbf{P}) + (\hat{\alpha}^* \cdot \mathbf{P}) \hat{\beta} + i \mathbf{P} \cdot (\hat{\alpha}^* \times \hat{\alpha})$$

$$\begin{aligned} \text{Tr} \hat{\rho} \hat{v}^* \hat{\sigma} \hat{v} = & \hat{\beta}^* \hat{\alpha} + \hat{\alpha}^* \hat{\beta} + \hat{\beta}^* \hat{\beta} \mathbf{P} + \hat{\alpha}^* (\hat{\alpha} \cdot \mathbf{P}) + (\hat{\alpha}^* \cdot \mathbf{P}) \hat{\alpha} - \\ & - \mathbf{P} \cdot (\hat{\alpha}^* \cdot \hat{\alpha}) - i \hat{\alpha}^* \times \hat{\alpha} + i \hat{\beta}^* (\hat{\alpha} \times \mathbf{P}) + i (\mathbf{P} \times \hat{\alpha}^*) \hat{\beta} \end{aligned}$$

Going from Nuclear & Magnetic Potential to Scattering Amplitudes

- The scattering potential $\vec{V} = \hat{\beta} + \vec{\hat{\alpha}} \cdot \vec{\hat{\sigma}}$

$$\hat{V}_N(\kappa) = \frac{2\pi\hbar^2}{m} \sum_{i,d} \exp(i\kappa \cdot \mathbf{R}_{id}) \hat{b}_{id}$$

$$\hat{V}_M(\kappa) = \left(\frac{2\pi\hbar^2}{m} \right) \left(\frac{\gamma e^2}{m_e c^2} \right) \hat{\sigma} \cdot \sum_i \exp(i\kappa \cdot \mathbf{r}_i) \left\{ \hat{\mathbf{r}} \times (\hat{\mathbf{s}}_i \times \hat{\mathbf{r}}) - \frac{i}{\hbar|\kappa|} \hat{\mathbf{r}} \times \hat{\mathbf{p}}_i \right\}$$

- The scattering amplitudes (nuclear & magnetic)

- Nuclear structure factor [fm]

- Magnetic structure factor [μ_B]

- Magnetic interaction vector [fm]

$$N(\vec{Q}) = \sum_j \bar{b}_j \exp(i\vec{Q} \cdot \vec{R}_j) \exp(-W_j)$$

$$\bar{M}(\vec{Q}) = \sum_j f_j(\vec{Q}) \bar{m}_j \exp(i\vec{Q} \cdot \vec{r}_{0j}) e^{-W_j}$$

$$\bar{M}_\perp(\vec{Q}) = \frac{\gamma e^2}{2m_e c^2} \vec{Q} \times \bar{M}(\vec{Q}) \times \vec{Q}$$

$$\frac{\gamma e^2}{2m_e c^2} = 2.695 [\text{fm} / \mu_B]$$

Nuclear & Magnetic Scattering & Polarisation Vector

- The Blume-Maleyev equations for a magnetic crystal

$$\sigma_i = (NN^*) + (\vec{M}_\perp \cdot \vec{M}_\perp^*) + (i\vec{P}_0 \cdot (\vec{M}_\perp^* \times \vec{M}_\perp)) + (\vec{P}_0 \cdot (\vec{M}_\perp N^* + \vec{M}_\perp^* N))$$

$$\begin{aligned} \vec{P}\sigma_i = & (\vec{P}_0 NN^*) + (-\vec{P}_0(\vec{M}_\perp \cdot \vec{M}_\perp^*) + \vec{M}_\perp(\vec{P}_0 \cdot \vec{M}_\perp^*) + \vec{M}_\perp^*(\vec{P}_0 \cdot \vec{M}_\perp)) + (-i(\vec{M}_\perp^* \times \vec{M}_\perp)) \\ & + (N\vec{M}_\perp^* + N^*\vec{M}_\perp - i(N\vec{M}_\perp^* - N^*\vec{M}_\perp) \times \vec{P}_0) \end{aligned}$$

2 TECHNIQUES

Polarimetric Neutron Scattering

- 1- Control incident polarisation vector
- 2- Scatter the neutron
 - Nuclear scalar amplitude (strong force)
 - Magnetic interaction Vector (dipolar)
- 3- Measure intensity and final polarisation vector

The neutron magnetic interaction vector

$$\vec{M}_\perp(\vec{Q}) = \frac{\gamma e^2}{2m_e c^2} \frac{\vec{Q} \wedge \vec{M}(\vec{Q}) \wedge \vec{Q}}{Q^2}$$

- effective part of the magnetic Fourier component
- $P = 2.695 \text{ fm}/\mu_B$

The elastic equations, Blume & Maleyev, 1963

- 2 equations

- cross-section

$$\sigma = \sigma_n + \sigma_m + \sigma_i$$

- final polarisation

$$\vec{P}_f \sigma = (\vec{P} \sigma)_n + (\vec{P} \sigma)_m + (\vec{P} \sigma)_i$$

- 3 parts

- nuclear n
- magnetic m
- interference i

The Nuclear part

- Nuclear cross section

$$\sigma_n = N^* N$$

- Unchanged polarisation

$$(\vec{P}\sigma)_n = \vec{P}_0 N^* N = \vec{P}_0 \sigma_n$$

The non chiral, magnetic part

- Magnetic cross section

$$\sigma_m = \vec{M}_\perp \cdot \vec{M}_\perp^*$$

- Changed polarisation
 - 2 components flipped
 - 1 component unflipped

$$(\vec{P}\sigma)_m = -\vec{P}_0 \sigma_m \{1 - (\hat{P}_0 \cdot \hat{M}_\perp) \hat{M}_\perp\}$$

and the chiral magnetic part

- x component even
- y component odd

$$\sigma_c = i \left(\vec{M}_\perp^* \times \vec{M}_\perp \right) \cdot \vec{P}_0$$

$$(\vec{P}\vec{\sigma})_c = -i \left(\vec{M}_\perp^* \times \vec{M}_\perp \right)$$

The nuclear-magnetic interference part

$$\sigma_i = \left(N \vec{M}_\perp^* + N^* \vec{M}_\perp \right) \cdot \vec{P}_0$$

$$(\vec{P}\vec{\sigma})_i = N \vec{M}_\perp^* + N^* \vec{M}_\perp$$

$$-i \left[\left(N \vec{M}_\perp^* - N^* \vec{M}_\perp \right) \times \vec{P}_0 \right]$$

- If N & M are in phase:
 - create polarisation along $M_\perp$
 - change cross section
- If N & M are in phase quadrature:
 - Rotate polarisation around $M_\perp$
 - no change in cross section

D3 in High-Field configuration

In Phase Nuclear-Magnetic Interference

$$\sigma = N N^* + \vec{M}_\perp \cdot \vec{M}_\perp^* + (N \vec{M}_\perp^* + N^* \vec{M}_\perp) \cdot \vec{P}_0$$

- ferromagnetic or paramagnetic under high field (10 Tesla)
- the change of cross section with incident polarisation is a very sensitive measurement of magnetic amplitudes
- maximum entropy Fourier mapping
- YCO_5 example

Phase quadrature in Cr_2O_3 :
no flipping ratio
but polarisation vector rotates

- Antiferro with zero propagation vector
- M/N in phase quadrature (opposite moments related by a center):

$$(N \vec{M}_\perp^* + N^* \vec{M}_\perp) = 0$$

- no change in cross section

$$\sigma_i = (N \vec{M}_\perp^* + N^* \vec{M}_\perp) \cdot \vec{P}_0 = 0$$

- rotation of polarisation around M

$$(\vec{P}\sigma)_i = N \vec{M}_\perp^* + N^* \vec{M}_\perp - i \left[(N \vec{M}_\perp^* - N^* \vec{M}_\perp) \times \vec{P}_0 \right]$$

- 180 domains: opposite rotation

First Polarisation Analysis
Moon, Riste, Koehler 1969

- Experimental layout
 - Magnetic crystal polariser
 - Magnetic crystal analyser
 - RF-flippers
 - adiabatic guide-fields all along

Guide Field Effect "Longitudinal" Polarisation Analysis

- Transverse components of scattered polarisation are not "guided", therefore not measured.
- It is impossible to distinguish a true depolarisation effect from a "trivial" rotation.
- By looking only at longitudinal component of polarisation it is difficult to see a small rotation.

Transverse component of polarisation H. Alperin, 1973

Zero Field Chamber, Okorokov 1978

Cryopad-I with Meissner Shields, Tasset 1989

- Access to large momentum transfer
- Variable temperature insert
- In/Out Spin Polarisation Vectors full control
 - Adiabatic rotators
 - Meissner shields
 - Larmor precession coils
 - Zero-field sample chamber

Cryopad-II and NSF at D3 , Tasset & al 1996

D3c with Cryopad-III and NSF - Lelièvre et al 2004

Cr₂O₃: 180° Magneto-electric Domains

$$\sigma_i = (N \vec{M}_\perp^* + N^* \vec{M}_\perp) \cdot \vec{P}_0$$

$$(\vec{P}_0)_i = N \vec{M}_\perp^* + N^* \vec{M}_\perp - i \left[(N \vec{M}_\perp^* - N^* \vec{M}_\perp) \times \vec{P}_0 \right]$$

- antiferro with magneto-electric effect
- zero propagation vector (N & M superposed in reciprocal space)
- N & M in phase quadrature (opposite moments related by a center of symmetry):
 - no change in cross section
 - rotation of polarisation around M
 - 180° domains: opposite rotation
- We need to unbalance domains & to measure both rotation and depolarisation
 - Use magneto-electric effect

Antiferromagnetic Cr form factor in Cr₂O₃, Brown et al. 1998

- 1) cooling in combined electric and magnetic field to get a majority domain
- 2) domain population from depolarisation
- 3) M/N from the polarisation rotation
- 4) free of secondary extinction

Covalent density in Cr_2O_3

- Standard 3d orbital refinement
- Maximum Entropy Fourier Difference Transformation

Inelastic Neutron Scattering

- Well defined excitations (phonons, magnons)
- Quantum fluctuations (quasi-elastic, spin gap)
- Hybrid excitations?

Inelastic measurements in CuGeO₃
 Regnault et al. 1998- IN20,IN14,IN22

INMI, inelastic nuclear-magnetic interference
 Maleyev, 1998

- generalized dyn. susceptibility
- Dispersive + i. Absorptive part
- Van Hove Correlation Function
- S±: mixed VHC Function

$$\langle A, B \rangle_{\omega} = i \int_{-\infty}^{\infty} dt e^{-i\omega t} \langle A(t), B(0) \rangle$$

$$\langle A, B \rangle_{\omega} = \langle A, B \rangle_{\omega}^{\prime} + i \langle A, B \rangle_{\omega}^{\prime\prime}$$

$$S_{AB}(\omega) = \chi \left[1 - e^{(-\eta)} \right]^{-1} \langle A, B \rangle_{\omega}^{\prime}$$

$$\langle N_{-Q}, \tilde{M}_Q \rangle_{\omega}^{\prime} = \langle N_{-Q}, \tilde{M}_Q \rangle_{\omega}^{\prime} = \langle \tilde{M}_{-Q}, N_Q \rangle_{\omega}^{\prime}$$

$$\sigma_i = (k_f / k_i) i \vec{S}_+ \cdot \vec{P}_0$$

$$(\vec{P}\sigma)_i = (k_f / k_i) \left\{ \vec{S}_+ + i \left[\vec{S}_- \times \vec{P}_0 \right] \right\}$$

- Strong motivation to measure small transverse components of polarisation because they are a signature of hybrid nuclear-magnetic excitations
- Needed:
 - 1) Cryopadum (combination of Meissner shields and mu-metal)
 - Large sample
 - Focusing geometry
 - Compactness
 - High Magnetic homogeneity, 1 Cryopad with better precision: Cryopadum project (ILL, CEA-G, JAERI-ASRC)
 - 2) high performance Neutron Spin Filter

Cryopadum for Inelastic Scattering

Transmission Neutron Spin Filter

- ^3He is a gas with a huge $1/v$ absorption cross section for thermal neutrons

Polarised ^3He spin filter

The neutron absorption cross section of ^3He is highly spin dependent...

$$\sigma_{\uparrow} = 0$$

$$\sigma_{\downarrow} = 6000 \text{ barns}$$

Anti-parallel neutron spins tend to be absorbed...
...while parallel ones tend to be transmitted.

Optical Pumping

Optical pumping : Principle

LKB

MEOP

Optical pumping : metastability exchange

A two-step process:

- Electronic optical pumping + polarisation transfer to the nucleus (hyperfine coupling)
- Metastability exchange collisions $\Rightarrow$ nuclear polarisation transfer to the $1S_0$

Atoms
 $P \sim 1$ millibar $\Leftrightarrow 1S_0$ density $\sim 3 \times 10^{16}$ atoms/cm³
 $(2^3S$ metastable density $\sim 3 \times 10^{10}$ atoms/cm³)

Laser
 For 1 Watt/cm² and typical absorptions of 1.4 %/cm
 $\Rightarrow 10^{17}$ polarised atoms/cm³/sec

} Typical pumping time ~ 1 sec.

MEOP (metastable exchange optical pumping)

- maximum 80% ^3He polarisation at 1 mbar
- high quantum efficiency
- Yb doped fibre laser (2W-5W-7W)
- (ideally 100 Joule/cm² are needed at 1 ang. wavelength)
 LKB, Mainz, ILL, NIST, HMI, ISIS
- rapid production of highly polarised gas at 1mbar pressure

Thickness & Quality factor

- Optimum thickness is for

$$n\alpha d = 2.2$$
- That is for example 3 bars at 9 cm at 1 Å
- requires polarisation preserving compressor unit PPC

Tyrex: MEOP-PP Volumetric Compression Unit for polarized 3He gas

- Non magnetic Titanium alloy
- Polarisation Preserving Compression
- Stroke volume 5.2 liters, displacement 55 cm
- Fast hydraulic translation (25 cm/ s)
- High precision in final position (2 μm)
- Typical flow rate: 45 mmole/hour

Optimised NSF Cells

- Cells with Silicon windows for reflectometry and SANS
- Up to 5 Bar Quartz cells for short wavelength on D3
- Also GE 180 cells

Remote-NSF for D3

Statistics on D3

3 CONCLUSION

- Spherical Neutron Polarimetry, SNP
 - Powerful vectorial method for magnetic interaction vector determination in antiferromagnets
 - Offers high magnetic sensitivity when nuclear-magnetic interference terms exist
 - In inelastic mode it has the potential to detect hybride nuclear-magnetic excitations.
- Neutron Spin Filter
 - The best present approximation to a spin sensitive detector working nicely at short wavelength.
 - Decouples spin from optics (i.e. the transmission is independent from the value of Q)